that vegetative cover is capable of reducing the amount of dust haze that the winds can pick up. Koval's results then lend support to the idea of vegetation on Mars, as is indicated by Sinton's spectrographic studies at the Lowell Observatory. The character of the seasonal and secular changes in the maria also support the interpretation of vegetation.

In conclusion:

1) The paper does reveal a considerable amount of Russian work.

2) Dr. Koval's paper is in need of some further elucidation for a mixed audience of American readers; especially a fuller explanation of the parameters of some of the graphs is needed. But even without it, the paper is worthy of acceptance.

3) This Russian paper certainly meets the requirement for originality.

4) Contrast measurements between maria and deserts have been made by others, but the author's method of analysis is clever and useful.

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Linear Theory of Three-Layered Shells with a Stiff Core

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THE object of this paper is a study of the general equilibrium equations of a three-layer shell with a stiff core in an orthogonal system of curvilinear coordinates. In the course of the analysis, the oscillations of an infinitely long three-layered cylindrical shell are considered also.

In deriving the equations the following assumptions are made. The faces and the core are made of different orthotropic materials, the orthotropic axes being mutually parallel. All the layers are of constant thickness. The core is incompressible in the transverse direction, and therefore the deflections of the middle surfaces of the faces will be the same. Displacements in the middle surfaces of the layers of the shell will be different for each of the layers. The usual Kirchhoff-Love hypotheses are assumed to hold for the faces, whereas for the core we shall establish a linear displacement law with respect to thickness. The expressions employed for the angles of inclination of the normal to the middle surface are more refined than in the theory of shallow shells.[1] It is assumed that the deformations always remain elastic.

First, we shall derive equilibrium equations for a shell of arbitrary shape acted upon by an arbitrary load and arbitrarily heated with respect to the thickness and over its surface. It is further assumed that the core is stiff, that is, capable of withstanding not only transverse shear but also loads parallel to its middle surface. A variational expression is given for the potential energy of the shell. This yields a system of equilibrium equations and the boundary conditions. As an example, equilibrium equations in terms of displacements are presented for a cylindrical shell.

Finally, equations are derived for the free oscillations of an infinitely long cylindrical shell, the core of which will withstand only transverse shear. An equation for determining the natural frequencies of the shell is derived also.

General Equations of the Problem

1. Displacements and Strains

As our surface of reference, we shall take the middle surface of the core. We shall assume that after deformation a rectilinear element of the core, normal to the thickness, is not distorted but remains straight. Then (see Fig. 1) the displacements in a surface equidistant from the middle surface will have the following form:

In the upper face ($c/2 \leqq z \leqq c/2 + t$):

$$u = u_1 + \left(z - \frac{c+t}{2}\right)\psi_1$$
$$v = v_1 + \left(z - \frac{c+t}{1}\right)\psi_2 \tag{1.1}$$

In the lower face $[-(c/2) - t \leqq z \leqq -(c/2)]$:

$$u = u_2 + \left(z + \frac{c+t}{2}\right)\psi_1$$
$$v = v_2 + \left(z + \frac{c+t}{2}\right)\psi_2 \tag{1.2}$$

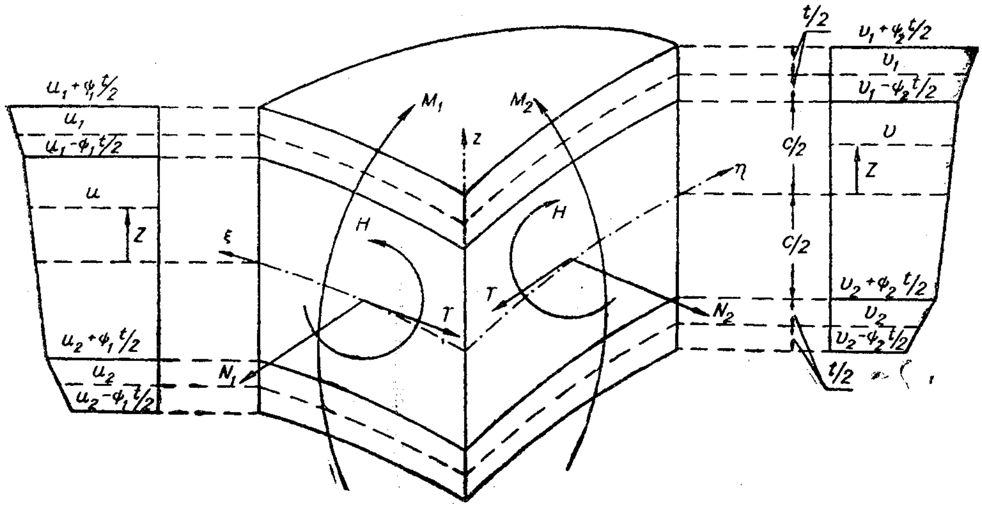

Fig. 1

Translated from Izvestiia Sibirskogo Otdeleniia Akademii Nauk SSSR (Bulletin of the Academy of Sciences of the USSR, Siberian Branch), No. 3, 12–24 (1962). Translated by Faraday Translations, New York.

In the filler $[-(c/2) \leqq z \leqq (c/2)]$:

$$u = \frac{u(c/2) + u(-c/2)}{2} + \frac{u(c/2) - u(-c/2)}{c} z \qquad v = \frac{v(c/2) + v(-c/2)}{2} + \frac{v(c/2) - v(-c/2)}{c} z \tag{1.3}$$

Here

$$\psi_1 = \frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{u}{R_1} \qquad \psi_2 = \frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{v}{R_2} \tag{1.4}$$

are the angles of inclination of the normal to the middle surface of the shell; A, B are Lamé parameters; ξ, η are orthogonal system of curvilinear coordinates in the middle surface of the core; u, v are displacements of a point on the surface along axes ξ, η; u_1, v_1, u_2, v_2 are displacements of a point on the middle surface of the upper and lower faces, respectively, along axes ξ, η; w is deflection, identical for all points lying on the same normal; R_1, R_2 are principal radii of curvature of the middle surface of the core; and c, t are thickness of the core and faces, respectively.

The normal and shearing strains[2] are

$$\begin{aligned} \epsilon_1 &= \frac{1}{A}\frac{\partial u}{\partial \xi} + \frac{1}{AB}\frac{\partial A}{\partial \eta} v - \frac{w}{R_1} & \epsilon_2 &= \frac{1}{B}\frac{\partial v}{\partial \eta} + \frac{1}{AB}\frac{\partial B}{\partial \xi} v - \frac{w}{R_2} \\ \gamma_{12} &= \frac{A}{B}\frac{\partial}{\partial \eta}\left(\frac{u}{A}\right) + \frac{B}{A}\frac{\partial}{\partial \xi}\left(\frac{v}{B}\right) \quad \gamma_{13} = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial \xi} & \gamma_{23} &= \frac{\partial v}{\partial z} - \frac{\partial w}{\partial \eta} \end{aligned} \tag{1.5}$$

since the shell is incompressible with respect to thickness, $\epsilon_2 = 0$.

2. Stresses

According to Hooke's law, the stresses are:
In the core:

$$\begin{gathered} \sigma_1 = \frac{E_1}{1 - \nu_{12}\nu_{21}} [\epsilon_1 + \nu_{21}\epsilon_2 - (\omega_1 + \nu_{21}\omega_2)T] \qquad \sigma_2 = \frac{E_2}{1 - \nu_{12}\nu_{21}} [\epsilon_2 + \nu_{12}\epsilon_1 - (\omega_2 + \nu_{12}\omega_1)T] \\ \tau_{12} = G_1\gamma_{12} \qquad \tau_{13} = G_2\gamma_{13} \qquad \tau_{23} = G_3\gamma_{23} \end{gathered} \tag{1.6}$$

In the faces:

$$\sigma_1' = \frac{E_1'}{1 - \nu_{12}'\nu_{21}'} [\epsilon_1' + \nu_{21}'\epsilon_2' - (\omega_1' + \nu_{21}'\omega_2')T] \qquad \sigma_2' = \frac{E_2'}{1 - \nu_{12}'\nu_{21}'} [\epsilon_2' + \nu_{12}'\epsilon_1' - (\omega_2' + \nu_{12}'\omega_1')T] \tag{1.7}$$

$$\tau_{12}' = G_1'\gamma_{12}'$$

Here E_1, E_2 are moduli of elasticity of the core; ν_{12}, ν_{21} are Poisson coefficients of the core; G_1, G_2, G_3 are shear moduli of the core; ω_1, ω_2 are coefficients of thermal expansion of the core; $T = T(\xi, \eta, z)$ is temperature; and the primed quantities relate to the faces.

3. Forces and Moments

We shall introduce the unit forces and moments per unit length in the following way (see Fig. 1):

$$\begin{aligned} N_1 &= N_{11} + N_{12} + N_{13} & M_1 &= M_{11} + M_{12} + M_{13} \\ N_2 &= N_{21} + N_{22} + N_{23} & M_2 &= M_{21} + M_{22} + M_{23} \\ T &= T_1 + T_2 + T_3 & H &= H_1 + H_2 + H_3 \end{aligned}$$

where

$$\begin{aligned} N_{11} &= \int_{c/2}^{c/2+t} \sigma_1' dz & N_{12} &= \int_{-c/2-t}^{-c/2} \sigma_1' dz & N_{13} &= \int_{-c/2}^{c/2} \sigma_1 dz \\ N_{21} &= \int_{c/2}^{c/2+t} \sigma_2' dz & N_{22} &= \int_{-c/2-t}^{-c/2} \sigma_2' dz & N_{23} &= \int_{-c/2}^{c/2} \sigma_2 dz \\ T_1 &= \int_{c/2}^{c/2+t} \tau_{12}' dz & T_2 &= \int_{-c/2-t}^{-c/2} \tau_{12}' dz & T_3 &= \int_{-c/2}^{c/2} \tau_{12} dz \\ M_{11} &= -\int_{c/2}^{c/2+t} \sigma_1' z dz & M_{12} &= -\int_{-c/2-t}^{-c/2} \sigma_1' z dz & M_{13} &= -\int_{-c/2}^{c/2} \sigma_1 z dz \\ M_{21} &= -\int_{c/2}^{c/2+t} \sigma_2' z dz & M_{22} &= -\int_{-c/2-t}^{-c/2} \sigma_2' z dz & M_{23} &= -\int_{-c/2}^{c/2} \sigma_2 z dz \\ H_1 &= -\int_{c/2}^{c/2+t} \tau_{12}' z dz & H_2 &= -\int_{-c/2-t}^{-c/2} \tau_{12}' z dz & H_3 &= -\int_{-c/2}^{c/2} \tau_{12} z dz \end{aligned} \tag{1.8}$$

Substituting in (1.8) the expressions for the stresses (1.6) and (1.7), taking into account Eqs. (1.1–1.5), and setting

$$U = \frac{u_1 + u_2}{2} \qquad V = \frac{v_1 + v_2}{2} \qquad \alpha = \frac{u_1 - u_2}{c + t} \qquad \beta = \frac{v_1 - v_2}{c + t}$$

we get

$$N_1 = 2B_1' \left[\frac{1}{A}\frac{\partial U}{\partial \xi} + \frac{1}{AB}\frac{\partial A}{\partial \eta} V - \frac{w}{R_1} + \nu_{21}' \left(\frac{1}{B}\frac{\partial V}{\partial \eta} + \frac{1}{AB}\frac{\partial B}{\partial \xi} U - \frac{w}{R_2}\right)\right] + B_1 \left[\frac{1}{A}\frac{\partial}{\partial \xi}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(V - \frac{t(c+t)}{4R_2}\beta\right) - \frac{w}{R_1} + \nu_{21}\left\{\frac{1}{B}\frac{\partial}{\partial \eta}\left(V - \frac{t(c+t)}{4R_2}\beta\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) - \frac{w}{R_2}\right\}\right] - 2B_1'(m_1' + \nu_{21}'m_2') - B_1(m_1 + \nu_{21}m_2)$$

$$N_2 = 2B_2' \left[\frac{1}{B}\frac{\partial V}{\partial \eta} + \frac{1}{AB}\frac{\partial B}{\partial \xi} U - \frac{w}{R^2} + \nu_{12}' \left(\frac{1}{A}\frac{\partial U}{\partial \xi} + \frac{1}{AB}\frac{\partial A}{\partial \eta} V - \frac{w}{R_1}\right)\right] + B_2 \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(V - \frac{t(c+t)}{4R_2}\beta\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) - \frac{w}{R_2} + \nu_{12}\left\{\frac{1}{A}\frac{\partial}{\partial \xi}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(V - \frac{t(c+t)}{4R_2}\beta\right) - \frac{w}{R_1}\right\}\right] - 2B_2'(m_2' + \nu_{12}'m_1') - B_2(m_2 + \nu_{12}m_1) \quad (1.9)$$

$$T = 2B_3' \left[\frac{1}{B}\frac{\partial U}{\partial \eta} + \frac{1}{A}\frac{\partial V}{\partial \xi} - \frac{1}{AB}\left(\frac{\partial A}{\partial \eta} U + \frac{\partial B}{\partial \xi} V\right)\right] + B_3 \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) + \frac{1}{A}\frac{\partial}{\partial \xi}\left(V - \frac{t(c+t)}{4R_2}\beta\right) - \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(U - \frac{t(c+t)}{4R_1}\alpha\right) - \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(V - \frac{t(c+t)}{4R_2}\beta\right)\right]$$

$$M_1 = -2D_1' \left[\frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right) + \nu_{21}'\left\{\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right)\right\}\right] - C_1' \left[\frac{1}{A}\frac{\partial \alpha}{\partial \xi} + \frac{1}{AB}\frac{\partial A}{\partial \eta}\beta + \nu_{21}'\left(\frac{1}{B}\frac{\partial \beta}{\partial \eta} + \frac{1}{AB}\frac{\partial B}{\partial \xi}\alpha\right)\right] - D_1 \left[\frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right) + \nu_{21}\left\{\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right)\right\}\right] + 2D_1'(n_1' + \nu_{21}'n_2') + D_1(n_1 + \nu_{21}n_2)$$

$$M_2 = -D_2' \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right) + \nu_{12}'\left\{\frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right)\right\}\right] - C_2' \left[\frac{1}{B}\frac{\partial \beta}{\partial \eta} + \frac{1}{AB}\frac{\partial B}{\partial \xi}\alpha + \nu_{12}'\left(\frac{1}{A}\frac{\partial \alpha}{\partial \xi} + \frac{1}{AB}\frac{\partial A}{\partial \eta}\beta\right)\right] - D_2 \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right) + \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right) + \nu_{12}\left\{\frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right) + \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right)\right\}\right] + 2D_2'(n_2' + \nu_{12}'n_1') + D_2(n_2 + \nu_{12}n_1)$$

$$H = -D_3' \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right) + \frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right) - \frac{1}{AB}\frac{\partial A}{\partial \eta}\left(\frac{1}{A}\frac{\partial w}{\partial \xi} + \frac{1}{R_1} U\right) - \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{1}{B}\frac{\partial w}{\partial \eta} + \frac{1}{R_2} V\right)\right] - \frac{1}{2} C_3' \left[\frac{1}{B}\frac{\partial \alpha}{\partial \eta} + \frac{1}{A}\frac{\partial \beta}{\partial \xi} - \frac{1}{AB}\left(\frac{\partial A}{\partial \eta}\alpha + \frac{\partial B}{\partial \xi}\beta\right)\right] - \frac{1}{2} D_3 \left[\frac{1}{B}\frac{\partial}{\partial \eta}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right) + \frac{1}{A}\frac{\partial}{\partial \xi}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right) - \frac{1}{AB}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1} U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi}\right) - \frac{1}{AB}\frac{\partial B}{\partial \xi}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2} V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta}\right)\right]$$

Here

$$B_1 = \frac{E_1 c}{1 - \nu_{12}\nu_{21}} \qquad B_2 = \frac{E_2 c}{1 - \nu_{12}\nu_{21}} \qquad B_1' = \frac{E_1' t}{1 - \nu_{12}'\nu_{21}'} \qquad B_2' = \frac{E_2' t}{1 - \nu_{12}'\nu_{21}'}$$

$$D_1 = \frac{E_1c^3}{12(1-\nu_{12}\nu_{21})} \qquad D_2 = \frac{E_2c^3}{12(1-\nu_{12}\epsilon_{21})} \qquad D_1' = \frac{E_1't^3}{12(1-\nu_{12}'\nu_{21}')} \qquad D_2' = \frac{E_2't^3}{12(1-\nu_{12}'\nu_{21}')}$$

$$C_1' = \frac{E_1't(c+t)^2}{2(1-\nu_{12}'\nu_{21}')} \qquad C_2' = \frac{E_2't(c+t)^2}{2(1-\nu_{12}'\nu_{21}')} \qquad C_3' = G_1't(c+t)^2$$

$$B_3 = G_1c \qquad B_3' = G_1't \qquad D_3 = G_1c^3/6 \qquad D_3' = G_1't^3/6$$

$$m_1' = (m_{11}' + m_{22}')/2 \qquad m_2' = (m_{12}' + m_{21}')/2 \qquad n_1' = (n_{11}' + n_{22}')/2 \qquad n_2' = (n_{12}' + n_{21}')/2 \quad (1.10)$$

$$m_{11}' = \frac{1}{t}\int_{c/2}^{c/2+t} \omega_1'Tdz \qquad m_{22}' = \frac{1}{t}\int_{-c/2-t}^{-c/2} \omega_1'Tdz \qquad m_1 = \frac{1}{c}\int_{-c/2}^{c/2} \omega_1Tdz$$

$$m_{12}' = \frac{1}{t}\int_{c/2}^{c/2+t} \omega_2'Tdz \qquad m_{21}' = \frac{1}{t}\int_{-c/2-t}^{-c/2} \omega_2'Tdz \qquad m_2 = \frac{1}{c}\int_{-c/2}^{c/2} \omega_2Tdz$$

$$n_{11}' = \frac{12}{t^3}\int_{c/2}^{c/2+t} \omega_1'zTdz \qquad n_{22}' = \frac{12}{t^3}\int_{-c/2-t}^{-c/2} \omega_1'zTdz \qquad n_1 = \frac{12}{c^3}\int_{-c/2}^{c/2} \omega_1zTdz$$

$$n_{12}' = \frac{12}{t^3}\int_{c/2}^{c/2+t} \omega_2'zTdz \qquad n_{21}' = \frac{12}{t^3}\int_{-c/2-t}^{-c/2} \omega_2'zTdz \qquad n_2 = \frac{12}{c^3}\int_{-c/2}^{c/2} \omega_2zTdz$$

The equilibrium equations and boundary conditions will be obtained by an energy method. Thus, first we shall find the variation of the potential energy of the composite shell.

4. Variation of Potential Energy of Shell

For the core we shall take into account the transverse shear energy. Then the variation of the potential energy of the shell is

$$\delta\Pi = \int_{a_1}^{a_2}\int_{b_1}^{b_2}\left\{\int_{-c/2}^{c/2}(\sigma_1\delta\epsilon_1 + \sigma_2\delta\epsilon_2 + \tau_{12}\delta\gamma_{12} + \tau_{13}\delta\gamma_{13} + \tau_{23}\delta\gamma_{23})dz + \int_{c/2}^{c/2+t}(\sigma_1'\delta\epsilon_1' + \sigma_2'\delta\epsilon_2' + \tau_{12}'\delta\gamma_{12}')dz + \int_{-c/2-t}^{-c/2}(\sigma_1'\delta\epsilon_1' + \sigma_2'\delta\epsilon_2' + \tau_{12}'\delta\gamma_{12}')dz - q_1\delta U - q_2\delta V - p\delta w\right\} d\xi d\eta$$

Here a_1, a_2, b_1, b_2 are coordinates of the shell edges in the direction of the axes ξ, η; and q_1, q_2, p are components of the external surface load along the axes ξ, η, z. Taking into account Eqs. (1.1–1.7 and 1.9) and integrating by parts, we get

$$\delta\Pi = \int_{a_1}^{a_2}\int_{b_1}^{b_2}\left\{\left[-\frac{\partial}{\partial\xi}\left\{\frac{1}{A}\left(N_1 - \frac{M_1+m_1^*}{R_1}\right)\right\} + \frac{1}{AB}\frac{\partial B}{\partial\xi}\left(N_2 + \frac{M_2+m_2^*}{R_1}\right) - \frac{\partial}{\partial\eta}\left\{\frac{1}{B}\left(T - \frac{H+m_{12}^*}{R_1}\right)\right\} - \right.\right.$$
$$\left.\frac{1}{AB}\frac{\partial A}{\partial\eta}\left(T - \frac{H+m_{12}^*}{R_1}\right) - G_2\frac{t}{R_1}\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1}U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial\xi} - \frac{\partial w}{\partial\xi}\right) - q_1\right]\delta U +$$
$$\left[-\frac{\partial}{\partial\eta}\left\{\frac{1}{B}\left(N_2 - \frac{M_2+m_2^*}{R_2}\right)\right\} + \frac{1}{AB}\frac{\partial A}{\partial\eta}\left(N_1 - \frac{M_1+m_1^*}{R_2}\right) - \frac{\partial}{\partial\xi}\left\{\frac{1}{A}\left(T - \frac{H+m_{12}^*}{R_2}\right)\right\} - \right.$$
$$\left.\frac{1}{AB}\frac{\partial B}{\partial\xi}\left(T - \frac{H+m_{12}^*}{R_2}\right) - G_3\frac{t}{R_2}\left(\frac{c+t}{c}\beta - \frac{t}{cR_2}V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial\eta} - \frac{\partial w}{\partial\eta}\right) - q_2\right]\delta V +$$
$$\left[-\frac{\partial}{\partial\xi}\left\{\frac{1}{A^2}\frac{\partial}{\partial\xi}(M_1+m_1^*) - \frac{1}{A^2B}\frac{\partial B}{\partial\xi}(M_2+m_2^*) + \frac{2}{A^2B}\frac{\partial A}{\partial\eta}(H+m_{12}^*) - C_2\left(\frac{t}{A}+c\right)\times\right.\right.$$
$$\left.\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1}U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial\xi} - \frac{\partial w}{\partial\xi}\right)\right\} - \frac{\partial}{\partial\eta}\left\{\frac{1}{B^2}\frac{\partial}{\partial\eta}(M_2+m_2^*) - \frac{1}{AB^2}\frac{\partial A}{\partial\eta}(M_1+m_1^*) + \right.$$
$$\left.\frac{2}{AB^2}\frac{\partial B}{\partial\xi}(H+m_{12}^*) - G_3\left(\frac{t}{B}+c\right)\left(\frac{c+t}{c}\beta - \frac{t}{cR_2}V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial\eta} - \frac{\partial w}{\partial\eta}\right)\right\} - 2\frac{\partial^2}{\partial\xi\partial\eta}\left\{\frac{1}{AB}(H+m_{12}^*)\right\} -$$
$$\left.\left(\frac{N_1}{R_1}+\frac{N_2}{R_2}\right) - p\right]\delta w + \left[-\frac{\partial}{\partial\xi}\left\{\frac{1}{A}\left[m_1^* - \frac{(c+t)^2}{4R_1}(N_1+n_1^*)\right]\right\} - \frac{\partial}{\partial\eta}\left\{\frac{1}{B}\left[m_{12}^* - \frac{(c+t)^2}{4R_1}(T+n_{12}^*)\right]\right\} + \right.$$
$$\frac{1}{AB}\frac{\partial B}{\partial\xi}\left[m_2^* - \frac{(c\times t)^2}{4R_1}(N_1+n_1^*)\right] - \frac{1}{AB}\frac{\partial A}{\partial\eta}\left[m_{12}^* - \frac{(c+t)^2}{4R_1}(T+n_{12}^*)\right] +$$
$$\left.G_2(c+t)\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1}U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial\xi} - \frac{\partial w}{\partial\xi}\right)\right]\delta\alpha + \left[-\frac{\partial}{\partial\eta}\left\{\frac{1}{B}\left[m_2^* - \frac{(c+t)^2}{4R_2}(N_2+n_2^*)\right]\right\} - \right.$$
$$\frac{\partial}{\partial\xi}\left\{\frac{1}{A}\left[m_{12}^* - \frac{(c+t)^2}{4R_2}(T+n_{12}^*)\right]\right\} + \frac{1}{AB}\frac{\partial A}{\partial\eta}\left[m_1^* - \frac{(c+t)^2}{4R_2}(N_1+n_1^*)\right] -$$

$$\frac{1}{AB}\frac{\partial B}{\partial \xi}\left[m_{12}^* - \frac{(c+t)^2}{4R_2}(T + n_{12}^*)\right] + G_3(c+t)\left(\frac{c+t}{c}\beta - \frac{t}{cR_2}V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta} - \frac{\partial w}{\partial \eta}\right)\Big]\delta\beta\Big\} d\xi d\eta +$$

$$\int_{a_1}^{a_2}\left\{\frac{1}{B}\left(T - \frac{H + m_{12}^*}{R_1}\right)\delta U + \frac{1}{B}\left(N_2 - \frac{M_2 + m_2^*}{R_2}\right)\delta V + \left[\frac{1}{B^2}\frac{\partial}{\partial \eta}(M_2 + m_2^*) + \frac{2}{B}\frac{\partial}{\partial \xi}\left\{\frac{1}{A}(H + m_{12}^*)\right\} - \right.\right.$$

$$\frac{1}{AB^2}\frac{\partial A}{\partial \eta}(M_1 + m_1^*) - G_3\left(\frac{t}{B} + c\right)\left(\frac{c+t}{c}\beta - \frac{t}{cR_2}V - \frac{t}{c}\frac{1}{B}\frac{\partial w}{\partial \eta} - \frac{\partial w}{\partial \eta}\right)\Big]\delta w - \frac{1}{B^2}(M_2 + m_2^*)\delta\frac{\partial w}{\partial \eta} +$$

$$\left.\frac{1}{B}\left[m_{12}^* - \frac{(c+t)^2}{4R_1}(T + n_{12}^*)\right]\delta\alpha + \frac{1}{B}\left[m_2^* - \frac{(c+t)^2}{4R_2}(N_2 + n_2^*)\right]\delta\beta\right\}_{b_1}^{b_2} d\xi +$$

$$\int_{b_1}^{b_2}\left\{\frac{1}{A}\left(N_1 - \frac{M_1 + m_1^*}{R_1}\right)\delta U + \frac{1}{A}\left(T - \frac{H + m_{12}^*}{R_2}\right)\delta V + \left[\frac{1}{A_2}\frac{\partial \xi}{\partial}(M_1 + m_1^*) + \frac{2}{A}\frac{\partial}{\partial \eta}\left\{\frac{1}{B}(H + m_{12}^*)\right\} - \right.\right.$$

$$\frac{1}{A^2B}\frac{\partial B}{\partial \xi}(M_2 + m_2^*) - G_2\left(\frac{t}{A} + c\right)\left(\frac{c+t}{c}\alpha - \frac{t}{cR_1}U - \frac{t}{c}\frac{1}{A}\frac{\partial w}{\partial \xi} - \frac{\partial w}{\partial \xi}\right)\Big]\delta w - \frac{1}{A^2}(M_1 + m_1^*)\delta\frac{\partial w}{\partial \xi} +$$

$$\left.\frac{1}{A}\left[m_1^* - \frac{(c+t)^2}{4R_1}(N_1 + n_1^*)\right]\delta\alpha + \frac{1}{A}\left[m_{12}^* - \frac{(c+t)^2}{4R_2}(T + n_{12}^*)\right]\delta\beta\right\}_{a_1}^{a_2} d\eta - \left\{\left[\frac{2}{AB}(H + n_{12}^*)\right]_{a_1}^{a_2}\right\}_{b_1}^{b_2} \quad (1.11)$$

where

$$n_1^* = \frac{2}{c+t}(M_{11} - M_{12}) - \frac{c}{c+t}N_{13} \qquad m_1^* = \frac{c+t}{2}(N_{11} - N_{12}) - \frac{c+t}{c}M_{13}$$

$$n_2^* = \frac{2}{c+t}(M_{21} - M_{22}) - \frac{c}{c+t}N_{23} \qquad m_2^* = \frac{c+t}{2}(N_{21} - N_{22}) - \frac{c+t}{c}M_{23} \quad (1.12)$$

$$n_{12}^* = \frac{2}{c+t}(H_1 - H_2) - \frac{c}{c+t}T_3 \qquad m_{12}^* = \frac{c+t}{2}(T_1 - T_2) - \frac{c+t}{c}H_3$$

Subject to the equilibrium conditions $\delta\Pi = 0$, the integrands must be satisfied for any value of δU, δV, δw, $\delta\alpha$, $\delta\beta$. The first integral of expression (1.11) gives the five equilibrium equations of the shell; the second and third integrals give the boundary conditions; the last term represents the concentrated forces at the corners of the shell.

The linear equations in Ref. 1 can be obtained from those given here by passing to a Cartesian coordinate system (whereupon $A = B = 1$) and neglecting terms with R_1 or R_2 in the denominator.

5. Equations of a Cylindrical Shell in Terms of Displacements

As a special case of the foregoing equations, we shall find the equilibrium equations of a cylindrical shell ($A = B = 1$, $R_2 = R$, $R_1 = \infty$). Using the force and moment Eqs. (1.9) and (1.12), we get the equations of a cylindrical shell in the form

$$a_1U_{xx} + a_2U_{yy} + a_3V_{xy}' - a_4t\beta_{xy} - a_5\frac{1}{R}w_x = \phi_1\; b_1U_{xy} + b_2V_{xx} + b_3V_{yy} - b_4\frac{1}{R^2}V - b_5t\alpha_{xy} -$$

$$b_6t\beta_{xx} - b_7t\beta_{yy} + b_8\frac{1}{R}\beta + b_9tw_{xxy} + b_{10}tw_{yyy} - b_{11}\frac{1}{R}w_y = \phi_2\; d_1\frac{1}{R}U_x - d_2tV_{xxy} - d_3tV_{yyy} + d_4\frac{1}{R}V_y +$$

$$d_5tR\alpha_{xxx} + d_6tR\alpha_{xyy} - d_7\alpha_x + d_8tR\beta_{xxy} + d_9tR\beta_{yyy} - d_{10}\beta_y - d_{11}R^2w_{xxxx} - d_{12}R^2w_{xxyy} - d_{13}R^2w_{yyyy} + d_{14}w_{xx} +$$

$$d_{15}w_{yy} - d_{16}\frac{1}{R^2}w = \phi_3 - K_1tV_{xy} + K_2R^2\alpha_{xx} + K_3R^2\alpha_{yy} - K_4\alpha + K_5R^2\beta_{xy} - K_6R^2w_{xxx} - K_7R^2w_{xyv} + K_8w_x =$$

$$\phi_4 - l_1tU_{xy} - l_2tV_{xx} - l_3tV_{yy} + l_4\frac{1}{R}V + l_5R^2\alpha_{xy} + l_6R^2\beta_{xx} + l_7R^2\beta_{xy} - l_8\beta - l_9R^2w_{xxy} - l_{10}R^2w_{vyy} + l_{11}w_y = \phi_5 \quad (1.13)$$

where

$$a_1 = 1 \qquad a_2 = \frac{2B_3' + B_3}{2B_1' + B_1} \qquad a_3 = a_2 + \frac{2B_1'\nu_{21}' + B_1\nu_{21}}{2B_1' + B_1}$$

$$a_4 = \frac{c+t}{4R}\cdot\frac{B_1\nu_{21} + B_3}{2B_1' + B_1} \qquad a_5 = a_3 - a_2$$

$$b_1 = \frac{2B_2'\nu_{21}' + B_2\nu_{12}}{2B_2' + B_2} + \frac{b_2}{b_3} \qquad b_2 = b_3\frac{B_3 + 2B_3'}{2B_2' + B_2}$$

$$b_3 = 1 + \frac{t^2}{12R^2} \qquad b_4 = \frac{G_3t^2}{c(2B_2' + B_2)} \qquad b_5 = \tfrac{1}{4}(b_7\nu_{12} + b_6)$$

$$b_6 = \frac{c+t}{3R}\,\frac{B_3}{2B_2' + B_2} \qquad b_7 = \frac{c+t}{3R}\,\frac{B_2}{2B_2' + B_2}$$

$$b_8 = \frac{G_3 t(c+t)}{c(2B_2' + B_2)} \qquad b_9 = b_{10}\left(b_1 + \frac{b_2}{b_3}\right) \qquad b_{10} = \frac{t}{12R} \qquad b_{11} = b_8 + 1$$

$$d_1 = b_1 - \frac{b_2}{b_3} \qquad d_2 = b_9 \qquad d_3 = b_{10} \qquad d_4 = b_{11}$$

$$d_5 = \frac{c+t}{12R}\cdot\frac{B_1}{2B_2' + B_2} \qquad d_6 = \tfrac{1}{4}(b_7\nu_{12} + 2b_6)$$

$$d_7 = \frac{G_2(c+t)^2}{c(2B_2' + B_2)} \qquad d_8 = d_6 \qquad d_9 = \tfrac{1}{4}b_7 \qquad d_{10} = \tfrac{3}{4}\frac{t}{R}b_7 + \left[\frac{(c+t)}{t}\right]b_8$$

$$d_{11} = \frac{t^2}{12R^2}\frac{B_1 + 2B_1'}{2B_2' + B_2} \qquad d_{12} = \frac{t^2}{6R^2}b_9 \qquad d_{13} = \frac{t^2}{12R^2}$$

$$d_{14} = d_7 \qquad d_{15} = \frac{G_3(c+t)^2}{c(2B_2' + B_2)} \qquad d_{16} = 1 \qquad K_1 = b_5$$

$$K_2 = \frac{(c+t)^2}{12R^2}\frac{B_1 + 6B_1'}{2B_2' + B_2} \qquad K_3 = \frac{(c+t)^2}{12R^2}\frac{B_3 + 6B_3'}{2B_2' + B_2}$$

$$K_4 = d_7 \qquad K_5 = \frac{(c+t)^2}{12R^2}\frac{B_1\nu_{21} + 6B_1'\nu_{21}' + B_3 + 6B_3'}{2B_2' + B_2}$$

$$K_6 = (t/R)d_5 \qquad K_7 = (t/R)d_6 \qquad K_8 = d_{15}$$

$$l_1 = b_5 \qquad l_2 = b_6 \qquad l_3 = b_7 \qquad l_4 = b_8 \qquad l_5 = K_4$$

$$l_6 = \frac{(c+t)^2}{12R^2}\cdot\frac{\left(1 + \frac{3}{4}\frac{t^2}{R^2}\right)B_3 + \left(1 + \frac{t^2}{12R^2}\right)B_3'}{2B_2' + B_{32}}$$

$$l_7 = \frac{(c+t)^2}{12R^2}\cdot\frac{\left(1 + \frac{3}{4}\frac{t^2}{R^2}\right)B_2 + \left(1 + \frac{t^2}{12R^2}\right)B_2'}{2B_2' + B_2}$$

$$l_8 = K_8 \qquad l_9 = (t/4R)(b_7 + 2b_6) \qquad l_{10} = (t/4R)b_7 \qquad l_{11} = d_{10}$$

Here the subscripts x, y indicate differentiation with respect to the corresponding variables. The right-hand sides of Eqs. (1.13) include temperature terms and components of the external load.

Oscillations of a Cylindrical Shell

For a cylindrical shell of infinite length we assume the following displacements:

In the upper face:

$$v = v_1 + \left(z - \frac{c+t}{2}\right)\left(v_1 + \frac{\partial w}{\partial \varphi}\right)\frac{1}{R} \tag{2.1}$$

In the lower face:

$$v = v_2 + \left(z + \frac{c+t}{2}\right)\left(v_2 + \frac{\partial w}{\partial \varphi}\right)\frac{1}{R} \tag{2.2}$$

For the core we shall have:

$$v = \frac{v\left(\frac{c}{2}\right) + v\left(-\frac{c}{2}\right)}{2} + \frac{v\left(\frac{c}{2}\right) - v\left(-\frac{c}{2}\right)}{c}z$$

$$= V - \frac{t(c+t)}{4}\frac{\beta}{R} + \left[\frac{c+t}{c}\beta - \frac{t}{c}\frac{1}{R}\left(V + \frac{\partial w}{\partial \varphi}\right)\right]z$$

The term β/R can be neglected, since it is of the order of $|t/R| \ll 1$. Then, writing out the first two terms of this expression, we get

$$\frac{v_1 + v_2}{2} - \frac{t}{4R}(v_1 - v_2) = \frac{1}{2}\left(1 - \frac{1}{2}\frac{t}{R}\right)v_1 + \frac{1}{2}\left(1 + \frac{1}{2}\frac{t}{R}\right) \times v_2 \approx \frac{v_1 + v_2}{2} = V$$

Accordingly, for the core:

$$v = V + \left[\frac{c+t}{c}\beta - \frac{t}{c}\frac{1}{R}\left(V + \frac{\partial w}{\partial \varphi}\right)\right]z \tag{2.3}$$

The normal and shearing strains will be:

$$\epsilon_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \varphi} - w\right)$$

$$\gamma_{23} = \frac{\partial v}{\partial z} - \frac{1}{R}\frac{\partial w}{\partial \varphi} \tag{2.4}$$

For an infinitely long shell $\epsilon_1 = \epsilon_z = 0$. The stresses are expressed by Hooke's law.

Having computed the variation of the potential energy of the shell

$$\delta\Pi = \int_0^{2\pi}\left\{\int_{-c/2}^{c/2}(\sigma_2\delta\epsilon_2 + \tau_{22}\delta\gamma_{23})dz + \int_{c/2}^{c/2+1}\sigma_2'\delta\epsilon_2'dz + \int_{-c/2-t}^{-c/2}\sigma_2'\delta\epsilon_2'dz - p\delta w - q\delta V\right\}Rd\varphi$$

using Eqs. (2.1–2.4), we get the equilibrium equations of the shell. Substituting inertia forces for the components of the external loads p and q:

$$q = -\frac{2\gamma' t + \gamma c}{g}\frac{\partial^2 V}{\partial t^2}$$

$$p = -\frac{2\gamma' t + \gamma c}{g}\frac{\partial^2 w}{\partial t^2}$$

where γ', γ are the densities of the material of the faces and core, respectively; and g is acceleration due to gravity; we get the following system of equations for the free oscillations of the shell (in the absence of heating):

$$\begin{gathered} w''' - \alpha_1 w' + \alpha_2 V'' - \alpha_3 V - \alpha_4 R\beta'' + \alpha_5 R\beta - \alpha_6 \ddot{V} = 0 \\ w''' - \theta_1 w' + \theta_2 V'' - \theta_3 V - \theta_4 R\beta'' + \theta_5 R\beta = 0 \\ w^{\mathrm{IV}} - \gamma_1 w'' + \gamma_2 w + \gamma_3 V''' - \gamma_4 V' - \gamma_5 R\beta''' + \gamma_6 R\beta + \gamma_7 \ddot{w} = 0 \end{gathered} \tag{2.5}$$

where

$$\alpha_1 = \frac{12R^2}{t^2}\left[1 + \frac{G_3 t(c+t)}{c(2B_2' + B_2)}\right]$$

$$\alpha_2 = 1 + (12R^2/t^2)$$

$$\alpha_3 = \frac{12R^2}{t^2}\frac{G_3 t^2}{c(2B_2' + B_2)}$$

$$\alpha_4 = \frac{B_2(c+t)}{t(2B_2' + B_2)} \qquad \alpha_5 = \alpha_1 - \frac{12R^2}{t^2}$$

$$\alpha_6 = \frac{12R^2}{t^2}\frac{R^2(2\gamma' t + \gamma c)}{g(2B_2' + B_2)} \tag{2.6}$$

$$\theta_1 = \frac{12R^2}{t^2}\frac{G_3 t(c+t)}{cB_2}$$

$$\theta_2 = 1 \qquad \theta_2 = [l/(c+t)]\theta_1$$

$$\theta_4 = \left(1 + \frac{c^2}{R^2}\right)\frac{6B_2'(c+t)}{tB_2} + \frac{c+t}{t}$$

$$\theta_5 = \theta_1 \qquad \gamma_1 = [(c+t)/t]\alpha_5 \qquad \gamma_2 = \alpha_2 - 1$$

$$\gamma_3 = 1 \qquad \gamma_4 = \alpha_1 \qquad \gamma_5 = \alpha_5$$

$$\gamma_6 = \gamma_1 \qquad \gamma_7 = \alpha_6$$

The quantities entering into (2.6) are represented by Eqs. (1.10). Primes denote a derivative with respect to the variable φ, dots a derivative with respect to time t.

Equations (2.5) can be obtained from Eqs. (1.13) by equating to zero $U,\alpha,T(x, y, z)$, and all derivatives with respect to x, and also substituting inertial forces for the components of the external load.

We shall give the solution of system (2.5) in the form

$$V = V_0 e^{i\omega t}\cos n\varphi \qquad \beta = \beta_0 e^{i\omega t}\cos n\varphi \qquad w = w_0 e^{i\omega t}\sin n\varphi \tag{2.7}$$

where ω is the natural frequency.

Substituting solution (2.7) in Eqs. (2.5), we get a homogeneous system of equations in V_0, β_0, and w_0. For nonzero values of V_0, β_0, and w_0 the determinant of this system must be equal to zero. Consequently, on expanding the determinant, we get a characteristic equation for determining the natural frequencies of the shell:

$$A_1\omega^4 - A_2\omega^2 - A_3 = 0$$

where

$$A_1 = -R\alpha_6^2(\theta_4 n^2 + \theta_1)$$

$$A_2 = R\alpha_6[(\theta_4 n^2 + \theta_1)(n^4 + \alpha_1 n^2 + \alpha_2 n^2 + \alpha_3 + \gamma_2) - n^2(n^2 + \theta_1)(\alpha_4 n^2 + \gamma_1) - (n^2 + \theta_3)(\alpha_4 n^2 + \alpha_5)]$$

$$A_3 = Rn^2(n^2 + \alpha_1)[(n^2 + \alpha_1)(\theta_4 n^2 + \theta_1) - (\alpha_4 n^2 + \alpha_5)(n^2 + \theta_1)] + Rn^2(\alpha_4 n^2 + \gamma_1)[(n^2 + \theta_1) \times (\alpha_2 n^2 + \alpha_5) - (n^2 + \theta_3)(n^2 + \alpha_1)] + R(n^4 + \alpha_1 n^2 + \gamma_2)[(\alpha_4 n^2 + \alpha_5)(n^2 + \theta_3) - (\theta_4 n^2 + \theta_1)(\alpha_2 n^2 + \alpha_3)]$$

If we assume simpler expressions for the displacements in the layers of the shell, neglecting the ratios v_1/R and v_2/R, then, in an analogous manner, we arrive at the following system of equations for the free oscillations:

$$\begin{gathered} w' - V'' + \bar{\alpha}_1 \ddot{V} = 0 \\ w''' - \bar{\theta}_1 w' - \bar{\theta}_2 R\beta'' + \bar{\theta}_3 R\beta = 0 \\ w^{\mathrm{IV}} - \bar{\gamma}_1 w'' + \bar{\gamma}_2 w - \bar{\gamma}_2 V' - \bar{\gamma}_3 R\beta''' + \bar{\gamma}_4 R\beta' + \bar{\gamma}_5 \ddot{w} = 0 \end{gathered} \tag{2.8}$$

Here

$$\bar{\alpha}_1 = (t^2/12R^2)\alpha_6 \qquad \bar{\theta}_1 = \bar{\theta}_3 = \theta_1$$

$$\bar{\theta}_2 = \frac{R^2}{R^2 + c^2}\theta_4 + \frac{c^2(c+t)}{t(R^2 + c^2)}$$

$$\bar{\gamma}_1 = [(c+t)^2/t^2]\alpha_3 \qquad \bar{\gamma}_2 = 12R^2/t^2$$

$$\bar{\gamma}_3 = \alpha_4 \qquad \bar{\gamma}_4 = [(c+t)^2/t^2]\alpha_3 \qquad \bar{\gamma}_5 = \alpha_6$$

Substituting solution (2.7) in Eqs. (2.8), we get the characteristic equation

$$\bar{A}_1\omega^4 - \bar{A}_2\omega^2 - \bar{A}_3 = 0$$

where

$$\bar{A}_1 = R\bar{\alpha}_1\bar{\gamma}_5(\bar{\theta}_1 + \bar{\theta}_2 n^2)$$

$$\bar{A}_2 = -R[(\bar{\theta}_1 + \bar{\theta}_2 n^2)(\bar{\alpha}_1 n^4 + \bar{\alpha}_1\bar{\gamma}_1 n^2 + \bar{\alpha}_1\bar{\gamma}_2 + \bar{\gamma}_5 n^2) - \bar{\alpha}_1 n^2(\bar{\theta}_1 + n^2)(\bar{\gamma}_1 + \bar{\gamma}_3 n^2)]$$

$$\bar{A}_3 = Rn^4[(\bar{\theta}_1 + \bar{\theta}_2 n^2)(\bar{\gamma}_1 + n^2) - (\bar{\theta}_1 + n^2)(\bar{\gamma}_1 + \bar{\gamma}_3 n^2)]$$

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Reviewer's Comment

This paper presents a definite contribution to the field of small deformations of sandwich shells. The research is based on the common assumptions accompanying small deformation theories, together with the simplest assumption possible regarding sandwich analysis, namely, that the core is rigid and carries only shear and extensional forces.

It is of interest to note that a closely related linear elastic analysis of a flat sandwich panel under very general conditions of temperature and load was presented by I. K. Ebcioglu as Aeronautical Systems Division TR 61-128 "Thermo-Elastic Equations for a Sandwich Panel under Arbitrary Temperature Distribution, Transverse Load and Edge Compression," 1961. This report considers an arbitrary temperature distribution in all three directions, a general transverse load, and edge compression of the panel. The core is assumed to be orthotropic, as are the faces of different thicknesses and materials. The author obtained general differential equations for the panel and formulated boundary conditions. Furthermore, the author reduced the five governing differential equations for the sandwich panel to two